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Comment on "No Primordial Magnetic Field from Domain Walls"

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Abstract

We comment on the recent preprint hep-ph/0007123 by M.B. Voloshin, claiming that domain walls are diamagnetic. We show that the results presented therein are based on an incorrect treatment of the zero mode contribution to the vacuum energy density. We also shown that the correct treatment leads to the conclusion that domain walls are ferromagnetic, and can generate a primordial magnetic field.

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In Ref. [1] we suggested that the spontaneous generation of uniform magnetic condensate in QED_3 [2], [3], [4] could give rise to ferromagnetic domain walls at the electroweak phase transition. Moreover, we suggested that these domain walls generate a magnetic field at the electroweak scale which can be relevant for the generation of the *primordial* magnetic field.

In a recent paper [5], Voloshin claims that the generation of primordial magnetic fields by domain walls is very unlikely. In particular, the author of Ref. [5] points out that massive $(2+1)$ dimensional fermions bound to a domain wall behave diamagnetically rather than ferromagnetically. In addition, Voloshin affirms that our previous claim of ferromagnetism is based on a incorrect formula in Ref. [2] for the magnetic field contribution to the fermionic vacuum free energy.

The aim of the present comment is twofold. First, we show that the results of [5] derive from an incorrect treatment of the fermionic zero mode contribution to the zero temperature vacuum energy density. Secondly, we show that the finite temperature calculation of Ref. [5] agrees with our previous findings [4], [6]. Moreover, if one takes into account the correct definition of an Abelian magnetic field in presence of a varying scalar field condensate, then it turns out that the classical magnetic energy is proportional to the area of the wall. This last result supports our previous proposal in Ref. [1].

Let us consider the zero temperature vacuum energy density due to massive planar fermions in presence of a constant magnetic field:

$$E_{\mathrm{vac}} = -\frac{e\,B}{2\,\pi} \sum_{n=0}^{\infty} \sqrt{2\,e\,B\,n + m^2} \; . \tag{1}$$

The infinite sum in Eq. (1) needs to be regularized. Among the possible gauge-invariant choices, we employed the Schwinger proper-time regularization scheme. However, any gauge invariant regularization gives physically sensible results. Following Ref. [5] we regularize the sum by means of the gauge invariant cut-off $\exp(-\epsilon\, E_n^2)$, where ϵ is the regulator parameter and E_n is the energy of the levels. We get:

$$E^{(r)}_{\mathrm{vac}}(B) = -\frac{e\,B}{2\pi} \sum_{n=0}^{\infty} \sqrt{2\,e\,B\,n + m^2}\; \exp(-\epsilon\, 2\,e\,B\,n - \epsilon\, m^2) \; . \tag{2}$$

To evaluate the sum in Eq. (2), Voloshin uses the Poisson's summation formula:

$$\sum_{n=0}^{\infty} f(n) = \int_{\delta}^{\infty} f(x) \sum_{n=-\infty}^{\infty} \delta(x-n)\, dx = \sum_{k=-\infty}^{\infty} \int_{\delta}^{\infty} f(x)\, \exp(2\,\pi\, i\, k\, x)\, dx \quad , \tag{3}$$

where δ is such that $-1 < \delta < 0$. Now, observing that the summand in Eq. (2) is non singular at $n=0$, Voloshin assumes that one can set $\delta = 0$. However it is easy to see that this last assumption is not valid. Indeed, if we consider the $n=0$ term in Eq. (3) :

$$I_\delta \equiv \int_{\delta}^{\infty} f(x)\, \delta(x)\, dx \; , \tag{4}$$

we see that $\lim_{\delta\to 0^-} I_\delta = f(0)$, while $\lim_{\delta \to 0^+} I_\delta = 0$. It is now evident that the limit $\delta \to 0$ is not harmless. Indeed, the Voloshin's procedure is equivalent to ignore the zero mode contribution.

The correct procedure can be obtained as follows. First one must isolate the zero mode contribution and, then, one can apply the Poisson's summation formula to the $n \geq 1$ modes. In this way, in the weak magnetic field region we get for the vacuum energy density:

$$
\begin{aligned}
E_{\rm vac}(B) - E_{\rm vac}(0) &= -\frac{eB}{4\pi}|m| + \frac{e^2\,B^2}{2\,\pi^3\,|m|}\sum_{p=0}^{\infty}\frac{(-1)^p\,\zeta(2\,p+2)\,\Gamma(\frac{3}{2})}{\Gamma(\frac{1}{2}-2\,p)}\left(\frac{e\,B}{\pi\,m^2}\right)^{2p} \\
&= -\frac{eB}{4\pi}|m| + \frac{e^2\,B^2}{24\,\pi\,|m|} + \dots\;. \qquad (5)
\end{aligned}
$$

This last equation agrees with the results obtained in Ref. [2], and differs from Eq. (14) of Ref. [5] in the negative term, linear in the magnetic field. Obviously, it is the linear term which gives rise to the spontaneous magnetic condensation in QED_3. We would like to note that, in the case of the thermal corrections, Voloshin correctly separates the zero mode contribution in the free energy calculation; therefore, it is surprising that this is not done also at zero temperature in [5].

Equation (5) shows that the results reported in Ref. [5], corrected to include the zero mode contributions, fully support the spontaneous generation of a uniform magnetic condensate in QED_3 with massive fermions. Moreover, it is worthwhile to stress that even the free energy higher temperature study of Ref. [5] is in agreement with the one reported in Ref. [4], [6]. Indeed, if one takes into account the missing linear term in the Voloshin's zero temperature energy density, it easy to see that Eq.(18) of Ref. [5] is in accordance with our previous conclusion that the thermal corrections, even at infinite temperature, do not modify the spontaneous generation of the magnetic condensate.

Let us now comment on the Voloshin's claim that the domain walls cannot be a source of the primordial magnetic field. This statement in [5] is based on the argument that the classical energy of the induced magnetic field is proportional to the volume of the box, while the contribution due to the fermion modes localized on the wall scales with the area of the wall. In this way Voloshin's total energy of the system is given by:

$$
E(B) = L^3\,\frac{B^2}{2} + L^2\,f(B)\;, \qquad (6)
$$

where L is the linear size of the system. It turns out that the above equation forgets completely the non Abelian nature of the induced magnetic field. Indeed, in the case of varying scalar field condensate the correct definition of Abelian electromagnetic field is given by the t'Hooft's Abelian projection [7]. Taking into account that in our model the Abelian part of the Abelian projected magnetic field is induced by the fermionic modes localized on the wall, it is easy to see that the Abelian projected magnetic field vanishes in the regions where the scalar condensate is constant. So that, in general the appropriate expression for the Abelian magnetic field can be different from zero only in the regions where the scalar condensate varies. In our model the magnetic field is localized in a region of the order of the wall thickness Δ. Thus Eq. (6) is replaced by:

$$
E(B) = L^2\,\Delta\,\frac{B^2}{2} + L^2\,f(B)\;. \qquad (7)
$$

which led to the conclusions of Ref. [1].

In conclusion, it is worthwhile to stress that in the realistic case where the domain wall

interacts with the plasma, the magnetic field penetrates into the plasma over a distance of the order of the penetration length λ which is about an order of magnitude greater than Δ. It follows that the estimate in Ref. [1] of the induced magnetic field at the electroweak scale $B^* \simeq 5\,10^{24}\,Gauss$ is reduced by a factor $\sqrt{\frac{\Delta}{\lambda}} \sim 0.3$ which, however, is still of cosmological interest.

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